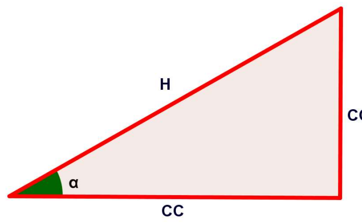
Trigonometria

Raons trigonomètriques d'un angle



$\sin(\alpha)=\frac{CO}{H}$	$\cos(\alpha)=\frac{CC}{H}$	$\tan(\alpha)=\frac{CO}{CC}=\frac{sin\alpha}{cos\alpha}$
$csec(\alpha)=\frac{1}{sin\alpha}$	$sec(\alpha)=\frac{1}{cos\alpha}$	$\mathrm{ctan}(\alpha)=\frac{1}{tan\alpha}$

Raons dels angles més utilitzats

α	$\sin(\alpha)$	$\cos(\alpha)$	$\tan(\alpha)$
$0\sim 0^o$	0	1	0
$\frac{\pi}{6}\sim 30^o$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{3}}{3}$
$\frac{\pi}{4}\sim 45^o$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	1
$\frac{\pi}{3}\sim 60^o$	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$
$\frac{\pi}{2}\sim 90^o$	1	0	$\nexists$

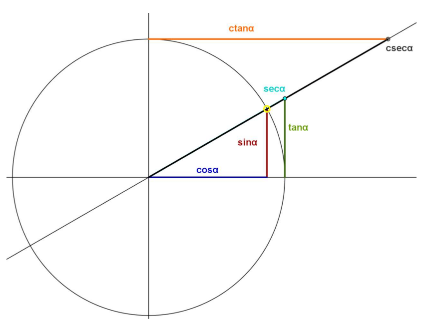
Igualtat fonamental de la trigonometria

$$\sin^2(\alpha)+\cos^2(\alpha)=1$$

Altres igualtats importants

$$1+tan^2\alpha=sec^2\alpha$$
$$1+ctan^2\alpha=csec^2\alpha$$

Circumferència Goniomètrica



Angle $\alpha+\beta$

$$\sin(\alpha+\beta)=sin\alpha\cdot cos\beta+sin\beta\cdot cos\alpha$$
$$\cos(\alpha+\beta)=cos\alpha\cdot cos\beta-sin\alpha\cdot sin\beta$$
$$\tan(\alpha+\beta)=\frac{tan\alpha+tan\beta}{1-tan\alpha\cdot tan\beta}$$

Angle $\alpha-\beta$

$$\sin(\alpha-\beta)=sin\alpha\cdot cos\beta-sin\beta\cdot cos\alpha$$
$$\cos(\alpha-\beta)=cos\alpha\cdot cos\beta+sin\alpha\cdot sin\beta$$
$$\tan(\alpha-\beta)=\frac{tan\alpha-tan\beta}{1+tan\alpha\cdot tan\beta}$$

Angle $\frac{\alpha}{2}$

$$\sin\left(\frac{\alpha}{2}\right)=\sqrt{\frac{1-cos\alpha}{2}}$$
$$\cos\left(\frac{\alpha}{2}\right)=\sqrt{\frac{1+cos\alpha}{2}}$$
$$\tan\left(\frac{\alpha}{2}\right)=\sqrt{\frac{1-cos\alpha}{1+cos\alpha}}$$

Producte a suma

$$\sin\alpha\cdot\sin\beta=\frac{1}{2}[\cos(\alpha-\beta)-\cos(\alpha+\beta)]$$
$$\cos\alpha\cdot\cos\beta=\frac{1}{2}[\cos(\alpha+\beta)+\cos(\alpha-\beta)]$$
$$\sin\alpha\cdot\cos\beta=\frac{1}{2}[\sin(\alpha+\beta)+\sin(\alpha-\beta)]$$

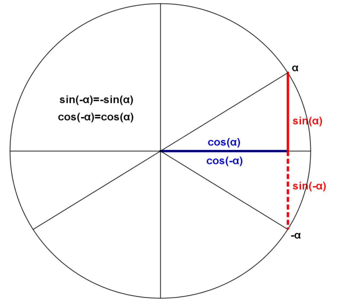
Suma a producte

$$sinA+sinB=2sin\frac{A+B}{2}\cos\frac{A-B}{2}$$
$$sinA-sinB=2\cos\frac{A+B}{2}sin\frac{A-B}{2}$$
$$\cos A+\cos B=2\cos\frac{A+B}{2}\cos\frac{A-B}{2}$$
$$\cos A-\cos B=-2sin\frac{A+B}{2}sin\frac{A-B}{2}$$

Angle 2α

$$\sin(2\alpha)=2sin\alpha\cdot cos\alpha$$
$$\cos(2\alpha)=cos^2\alpha-sin^2\alpha$$
$$tag(2\alpha)=\frac{2tag(\alpha)}{1-tag^2(\alpha)}$$

Reducció al 1r octant



$\sin\left(\frac{\pi}{2}-\alpha\right)=cos(\alpha)$ $\cos\left(\frac{\pi}{2}-\alpha\right)=sin(\alpha)$	$\sin\left(\frac{\pi}{2}+\alpha\right)=sin(\alpha)$ $\cos\left(\frac{\pi}{2}+\alpha\right)=-cos(\alpha)$
$\sin(\pi-\alpha)=\sin(\alpha)$ $\cos(\pi-\alpha)=-cos(\alpha)$	$\sin(\pi+\alpha)=-\sin(\alpha)$ $\cos(\pi+\alpha)=-cos(\alpha)$
$\sin\left(\frac{3\pi}{2}-\alpha\right)=-cos(\alpha)$ $\cos\left(\frac{3\pi}{2}-\alpha\right)=-sin(\alpha)$	$\sin\left(\frac{3\pi}{2}+\alpha\right)=cos(\alpha)$ $\cos\left(\frac{3\pi}{2}+\alpha\right)=sin(\alpha)$

Teorema del sinus

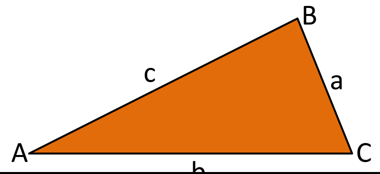
$$\frac{sinA}{a}=\frac{sinB}{b}=\frac{sinC}{c}$$
$$\frac{a}{sinA}=\frac{b}{sinB}=\frac{c}{sinC}$$

Teorema del cosinus

$$a^2=b^2+c^2-2bc\cos A$$
$$b^2=a^2+c^2-2ac\cos B$$
$$c^2=a^2+b^2-2ab\cos C$$

Fórmula d'Heron

$$p=\frac{a+b+c}{2}$$
$$A=\sqrt{p\cdot(p-a)\cdot(p-b)\cdot(p-c)}$$



Complexos C

Forma binòmica

$z=a+bi$ $\quad Re(z)=a\in R,\ Img(z)=b\in R$

$R\subset C$ ja que si $a\in R\rightarrow a=a+0i\in C$

Suma de complexos

$$\left.\begin{matrix}z_1=a_1+b_1i\\ z_2=a_2+b_2i\end{matrix}\right\}\rightarrow z_1+z_2=(a_1+a_2)+(b_1+b_2)i$$

Producte d'un escalar per un complex

$$\left.\begin{matrix}z=a+bi\\ \lambda\in R\end{matrix}\right\}\rightarrow\lambda\cdot z=\lambda a+\lambda bi$$

Producte de complexos

$$\left.\begin{matrix}z_1=a_1+b_1i\\ z_2=a_2+b_2i\end{matrix}\right\}\rightarrow z_1\cdot z_2=(a_1a_2-b_1b_2)+(a_1b_2+a_2b_1)i$$

Propietats de la suma

- $si\ z,w\in C\rightarrow z+w\in C$
- $z+w=w+z\ \forall z,w\in C$
- $(z+w)+u=z+(w+u)=z+w+u\ \forall z,w,u\in C$
- $z+0=0+z=z\ \forall z\in C, 0=0+0i$
- $z+(-z)=0\ \forall z\in C, -z=-a-bi$

Propietats del producte per un escalar

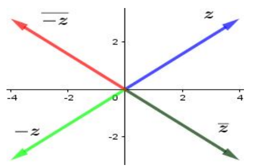
- $Si\ \alpha,\beta\in R\ i\ z\in C\rightarrow(\alpha+\beta)\cdot z=\alpha\cdot z+\beta\cdot z$
- $Si\ \alpha\in R\ i\ z_1,z_2\in C\rightarrow\alpha\cdot(z_1+z_2)=\alpha\cdot z_1+\alpha\cdot z_2$
- $Si\ \alpha,\beta\in R\ i\ z\in C\rightarrow(\alpha\cdot\beta)\cdot z=\alpha\cdot(\beta\cdot z)$
- $Si\ z\in C\rightarrow 1\cdot z=z$

Propietats del producte

- $si\ z,w\in C\rightarrow z\cdot w\in C$
- $z\cdot w=w\cdot z\ \forall z,w\in C$
- $(z\cdot w)\cdot u=z\cdot(w\cdot u)=z\cdot w\cdot u\ \forall z,w,u\in C$
- $z\cdot 1=1\cdot z=z\ \forall z\in C, 1=1+0i$
- $z\cdot\frac{1}{z}=1\ \forall z\in C^*$

Conjugat d'un complex

Si $z=a+bi$ el conjugat és $\bar{z}=a-bi$



Propietats del conjugat

$z\in C\rightarrow\bar{\bar{z}}=z$

$z\in C\rightarrow\overline{-z}=-\bar{z}$

$z,w\in C\rightarrow\overline{z\pm w}=\bar{z}\pm\bar{w}$

$z\in C\rightarrow Img(z)=\frac{z-\bar{z}}{2i}$

$z\in C\rightarrow Re(z)=\frac{z+\bar{z}}{2}$

$z,w\in C\rightarrow\overline{z\cdot w}=\bar{z}\cdot\bar{w}$

$z,w\neq 0\in C\rightarrow\frac{z}{w}=\frac{z\cdot\bar{w}}{w\cdot\bar{w}}$

$z,w\neq 0\in C\rightarrow\overline{\left(\frac{z}{w}\right)}=\frac{\bar{z}}{\bar{w}}$

Mòdul d'un complex

$Si\ z=a+b\,i\in C\ definim\quad |z|=+\sqrt{a^2+b^2}$

Propietats del mòdul

$Si\ z,w\in C\rightarrow|z+w|\leq|z|+|w|$

$Si\ z,w\in C\rightarrow|z\cdot w|=|z|\cdot|w|$

$Si\ z,w\neq 0\in C\rightarrow\left|\frac{z}{w}\right|=\frac{|z|}{|w|}$

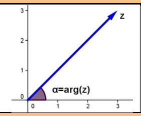
$Si\ z\in C\rightarrow z\cdot\bar{z}=|z|^2$

Forma polar

Argument d'un complex

Si $z=a+bi$ definim arg(z) com l'angle que determina el semieix positiu X amb el número complex z medit en sentit antihorari.

- Representem el complex $z=a+bi$
- Calculem els angles $\alpha\in[0,2\pi]$ que acompleixen $tan\alpha=\frac{b}{a}$
- Hi ha dos angles, elegim el que correspon a la representació.



$$z=|z|_\alpha$$

Forma trigonomètrica

$$\arg(z)=\alpha\rightarrow z=|z|(cos\alpha+i\cdot sin\alpha)$$

Multiplicació en forma polar

$$z=|z|_\alpha,\ w=|w|_\beta\rightarrow z\cdot w=(|z|\cdot|w|)_{\alpha+\beta}$$

Potenciació en forma polar

$$z=|z|_\alpha,\ n\in N\rightarrow z^n=\left(|z|^n\right)_{n\alpha}$$

Divisió en forma polar

$$z=|z|_\alpha,\ w=|w|_\beta\ on\ w\neq 0\rightarrow\frac{z}{w}=\left(\frac{|z|}{|w|}\right)_{\alpha-\beta}$$

Fórmula de Moivre

$$(cos\alpha+isin\alpha)^n=\cos(n\alpha)+isin(n\alpha)$$

Radicació en forma polar

$$Si\ \ n\in N^*\rightarrow\sqrt[n]{z}=\left(\sqrt[n]{|z|}\right)_{\frac{\arg(z)}{n}+\frac{2\pi k}{n}}\ on\ k=0,1,2,3,\ldots,(n-1)$$

$\sum vf$