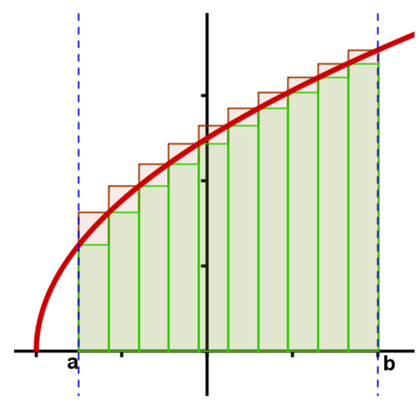
Integral definida $f\colon [a,b] \longrightarrow R$ contínua

$\Sigma\nu f$



Suma superior

$$U(f,P) = \sum_{i=1}^{n} M_i \cdot (x_i - x_{i-1})$$

$$M_i = \underset{x\in[x_{i-1},x_i]}{màxim}[f(x)]$$

Integral superior

$$\overline{\int_a^b} f(x)\,dx = \lim_{\|P\|\to 0} U(f,P)$$

Suma inferior

$$L(f,P) = \sum_{i=1}^{n} m_i \cdot (x_i - x_{i-1})$$

$$m_i = \underset{x\in[x_{i-1},x_i]}{mínim}[f(x)]$$

Integral inferior

$$\underline{\int_a^b} f(x)\,dx = \lim_{\|P\|\to 0} L(f,P)$$

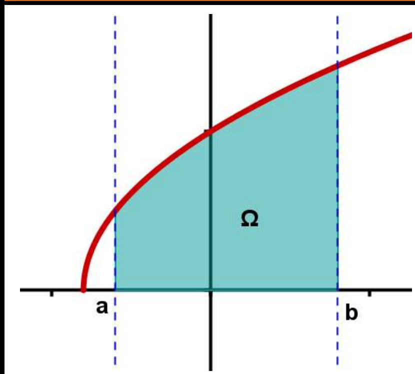
Funció integrable Riemann (IR)

$$f(x) \text{ és } IR \Leftrightarrow \underline{\int_a^b} f(x)\,dx = \overline{\int_a^b} f(x)\,dx$$

Integral definida

$$\int_a^b f(x)\,dx$$

Propietats de la integral definida



$$m\cdot(b-a) \le \int_a^b f(x)\,dx \le M\cdot(b-a) \text{ on } m = \underset{x\in[a,b]}{mínim}[f(x)] \text{ i } M = \underset{x\in[a,b]}{màxim}[f(x)]$$

$$\int_c^c f(x)dx = 0 \quad \forall c \in [a,b]$$

$$\int_a^b f(x)\,dx = -\int_b^a f(x)\,dx$$

$$\int_a^b f(x)\,dx = \int_a^c f(x)\,dx + \int_c^b f(x)\,dx \quad \forall c \in [a,b]$$

$$\int_a^b [f(x) \pm g(x)]\,dx = \int_a^b f(x)\,dx \pm \int_a^b g(x)\,dx$$

$$\int_a^b k\cdot f(x)\,dx = k\cdot\int_a^b f(x)\,dx$$

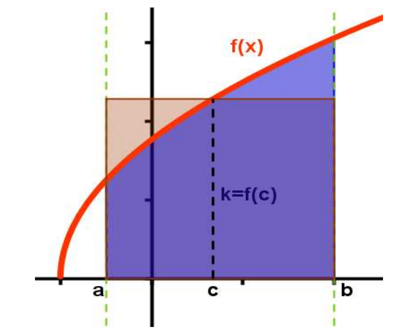
Partició d'un interval $[a,b]$ $\quad P = \{a = x_0, x_1, x_2, \ldots, x_n = b\} \quad \|P\| = màx(|x_i - x_{i-1}|) \;\; i = 1,2,3,\ldots,n$

Si $f\colon [a,b] \longrightarrow R$ contínua i positiva $\rightarrow \int_a^b f(x)\,dx = àrea(\Omega)$

Teorema del valor mitjà del càlcul integral

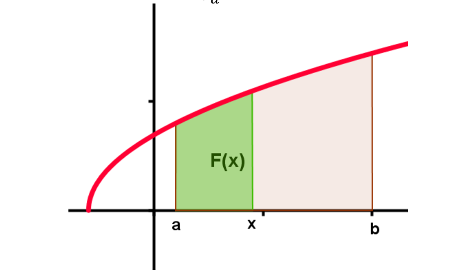
$$f\colon [a,b] \longrightarrow R \text{ integrable } \longrightarrow \exists k \in [m,M] \,/\, \int_a^b f(x)\,dx = k\cdot(b-a)$$

$$f\colon [a,b] \longrightarrow R \text{ contínua } \longrightarrow \exists c \in [a,b] \,/\, \int_a^b f(x)\,dx = f(c)\cdot(b-a)$$



Funció àrea

$$F(x) = \int_a^x f(t)\,dt$$



Teorema fonamental del càlcul integral

$F(x)$ és derivable i $F'(x) = f(x)$

Regla de Barrow

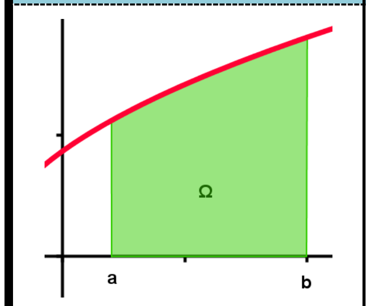
Si $G(x)$ és una primitiva d'$f(x) \longrightarrow \int_a^b f(x)\,dx = G(b) - G(a)$

Aplicacions de la integral definida

Càlcul d'àrees

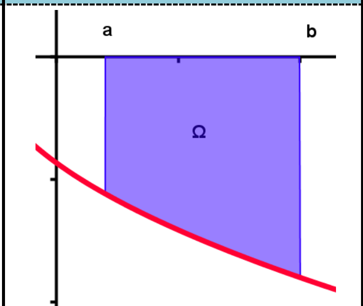
$$\Omega\begin{cases} f\colon [a,b] \longrightarrow R \text{ contínua i positiva} \\ x = a \\ x = b \\ y = 0 \;\; (eix\ X) \end{cases}$$

$$àrea(\Omega) = \int_a^b f(x)\,dx$$



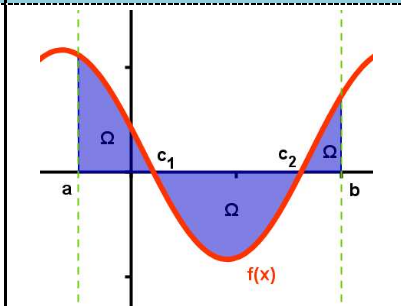
$$\Omega\begin{cases} f\colon [a,b] \longrightarrow R \text{ contínua i negativa} \\ x = a \\ x = b \\ y = 0 \;\; (eix\ X) \end{cases}$$

$$àrea(\Omega) = \left|\int_a^b f(x)\,dx\right| = -\int_a^b f(x)\,dx$$



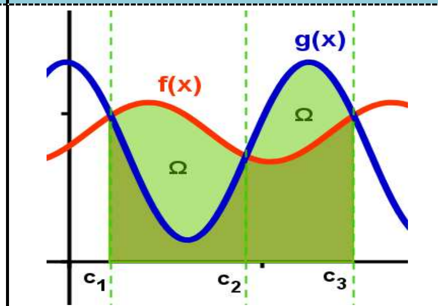
$$\Omega\begin{cases} f\colon [a,b] \longrightarrow R \text{ contínua} \\ \text{talla l'eix}X\ (f(x) = 0) \text{ en } c_1 \text{ i } c_2 \\ x = a \\ x = b \\ y = 0 \;\; (eix\ X) \end{cases}$$

$$àrea(\Omega) = \int_a^{c_1} f(x)\,dx + \left|\int_{c_1}^{c_2} f(x)\,dx\right| + \int_{c_2}^b f(x)\,dx$$



$$\Omega\begin{cases} f,g\colon [a,b] \longrightarrow R \text{ contínues} \\ f(x) \text{ i } g(x) \text{ es tallen } (f(x) = g(x)) \\ \text{en } c_1, c_2 \text{ i } c_3 \end{cases}$$

$$àrea(\Omega) = \int_{c_1}^{c_2} [f(x) - g(x)]\,dx + \int_{c_2}^{c_3} [g(x) - f(x)]\,dx$$



Càlcul de volums

$$\Omega\begin{cases} f\colon [a,b] \longrightarrow R \text{ contínua i positiva} \\ x = a \\ x = b \\ y = 0 \;\; (eix\ X) \end{cases}$$ al girar al voltatnt de l'eix X

$$V = \pi\cdot\int_a^b [f(x)]^2\,dx$$